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Proportion test statistic

Testing population proportion is a hypothesis testing procedure used to assess whether or not a sample from a population represents the true proportion of the entire population. Testing a sample population proportion is a widely used statistical method with various applications across different fields. The main purpose of testing a sample population proportion is to make inferences about an entire population based on the sample information. Testing a sample population proportion helps to determine whether an observed sample proportion is significantly different from a hypothesized population proportion. The following are some common uses of population proportion: Marketing research: To determine if a certain proportion of customers prefer one product compared to another. Quality control: In manufacturing, population proportion tests can be used to test/check if the proportion of defective items in a production batch exceeds an acceptable threshold. Medical research: To test the efficacy of a new treatment by comparing the proportion of patients who recover using the new treatment versus a standard treatment. Political polling: To estimate the proportion of voters supporting a particular candidate or policy. Social sciences: To examine the prevalence of certain behaviors or attitudes in a population. Business: Testing customer satisfaction rates, conversion rates in A/B testing for websites, or employee retention rates. Public health: Estimating vaccination rates, disease prevalence, or the effectiveness of public health campaigns. Education: Assessing the proportion of students meeting certain academic standards or the effectiveness of new teaching methods. Psychology: Evaluating the proportion of individuals exhibiting certain behaviors or responses in experiments. Environmental science: Measuring the proportion of samples that exceed pollution thresholds. There are two types of population proportion tests. One-sample z-test for proportion: One-sample proportion tests are used when comparing a sample proportion to a known or hypothesized population proportion. Two-sample z-test for proportions: Two-sample proportion tests are used when comparing proportions from two independent samples. The following are assumptions and considerations when testing population proportion: The sample should be randomly selected and representative of the population. The sample size (number of observations in the sample) should be large enough (typically np and $n(1-p)$ should both be greater than 5, where n is the sample size and p is the proportion). For two-sample tests, the samples should be independent of each other. Interpretation: The results of these tests are typically interpreted using p-values or confidence intervals, allowing researchers to make statistical inferences about the population based on the sample data. By using tests for population proportions, researchers and professionals can make data-driven decisions, validate hypotheses, and gain insights into population characteristics across a wide range of fields and applications. Suppose, a random sample is drawn and the population proportion (say) $\hat{p}$ is measured and $n\hat{p}\ge 5$, $n\hat{q}\ge5$, the distribution of $\hat{p}$ is approximately normal with $\mu_{\hat{p}} =p$ and $\sigma_{\hat{p}}=\sqrt{\frac{pq}{n}}$. Also, suppose that one of the possible null hypotheses of the following form, when testing a claim about a population proportion is: $H_o: p=p_o$$H_o:p\ge p_o$$H_o\le p_o$ For simplicity, we will assume the null hypothesis $H_o:p=p_o$. The standardized test statistics for a one-sample proportion test is \begin{align*}Z&=\frac{\hat{p} – \mu_{\hat{p}}}{\sigma_{\hat{p}}}\\&=\frac{\hat{p} -p_o }{\sqrt{\frac{p_oq_o}{n}}}\end{align*} This random variable will have a standard normal distribution. Therefore, the standard normal distribution will be used to compute critical values, regions of rejection, and p-values, as we use it to test a mean using a large sample. A computer chip manufacturer tests microprocessors coming off the production line. In one sample of 577 processors, 37 were found to be defective. The company wants to claim that the proportion of defective processors is only 4%. Can the company claim be rejected at the $\alpha = 0.01$ level of significance? Solution: The null and alternative hypotheses for testing the one-sample population proportion will be $H_o:p=0.04$$H_1:pe 0.04$ By focusing on the alternative hypothesis symbol (e), the test is two-tailed with $p_o=0.04$. The $\hat{p} = \frac{37}{577} \approx 0.064$. the standardized test statistics is \begin{align*}Z &= \frac{\hat{p} – p_o}{\sqrt{\frac{p_oq_o}{n}}}\\&=\frac{0.064 – 0.04}{\sqrt{\frac{(0.04)(0.96)}{577}}}\\&=\frac{0.024}{0.008}\approx 3.0\end{align*} Looking up $Z=3.00$ in the standard normal table (area under the standard normal curve), we get a value of 0.9987. Therefore, $P(Z\ge 3.00) = 1-0.9987) = 0.0013$. Note that the test is two-tailed, the p-value will be twice this amount or 0.0026. Since the p-value (0.0026) is less than the level of significance (0.01), that is $0.0025 < 0.01$ (p-value < level of significance), we will reject the company's claim. It means that the proportion of defective processors is not 4%, it is either less than 4% or more than 4%. An opinion poll of 1010 randomly chosen/selected adults finds that only 47% approve of the president's job performance. The president's political advisors want to know if this is sufficient data to show that less than half of adults approve of the president's job performance using a 5% level of significance. Solution: The null and alternative hypothesis of the problem above will be $H_o:p\ge 0.50$$H_1:p< 0.50$ By focusing on the alternative hypothesis symbol (Lesson 62 A hypothesis test for a proportion involves testing a claim about a population proportion (P) using sample data. Here's a step-by-step guide to performing a one-sample z-test for a proportion, which is the most common method for this type of test. When to Use This Analysis The approach described in this lesson is appropriate when the following conditions are met: The population size is at least 20 times as big as the sample size. (This condition is required to justify using an approximate formula to compute the standard error of the sampling distribution.) Before proceeding with a hypothesis test, ensure that these conditions are met. To test any hypothesis, the same five-step procedure is used: (1) state the hypotheses, (2) choose the significance level, (3) compute the test statistic, (4) find the P-value, and (5) interpret results. Here, we apply the general procedure to hypothesis tests of proportions. State the Hypotheses Every hypothesis test requires the analyst to state a null hypothesis and an alternative hypothesis . The hypotheses are stated in such a way that they are mutually exclusive. That is, if one is true, the other must be false, as shown below. Null hypothesis (H0): The population proportion equals P0. H0: P = P0 Alternative hypothesis (Ha): The population proportion differs from P0 in one of three possible ways. Ha: P ≠ P0 (Two-tailed test checking for any difference from P1) Ha: P > P0 (One-tailed test checking if P is greater than P1) Ha: P < P0 (One-tailed test checking if P is less than P1) where P is the true population proportion and P0 is the hypothesized population proportion. Choose the Significance Level The significance level is the probability of rejecting the null hypothesis when it is actually true. Often, researchers choose significance levels equal to 0.01, 0.05, or 0.10. Compute the Test Statistic Use the one-sample z-test to determine whether the hypothesized population proportion differs significantly from the observed sample proportion. The test statistic is a z-score (z) defined by the following equation. z = (p - P0) / SD; where P0 is the hypothesized value of population proportion in the null hypothesis, p is the sample proportion, and SD is the standard deviation of the sampling distribution. When the population size is at least 20 times bigger than the sample size, the standard deviation of the sampling distribution (SD) can be computed from this formula: SD = sqrt[P0 * (1 - P0) / n] where P0 is the hypothesized value of population proportion in the null hypothesis, and n is the sample size. Find the P-Value The P-value is the probability of observing a sample statistic as extreme as the z-score test statistic. To assess the probability associated with the z-score, use an online calculator, a graphing calculator, or a normal distribution statistical table. (See sample problems at the end of this lesson for examples of how this is done with Stat Trek's Normal Distribution Calculator.) Interpret Results If the sample findings are unlikely, given the null hypothesis, the researcher rejects the null hypothesis. This involves comparing the P-value to the significance level, and rejecting the null hypothesis when the P-value is less than the significance level. In this section, two hypothesis testing examples illustrate how to conduct a hypothesis test of a proportion. The first problem involves a a two-tailed test; the second problem, one-tailed test. As you probably noticed, the process of testing a hypothesis about a proportion can be complex. Stat Trek's Sample Size Calculator can do the same job quickly and easily. When you need to test a hypothesis, consider using the Sample Size Calculator. The calculator is free. It can found in the Stat Trek main menu under the Stat Tools tab. Or you can tap the button below. Sample Size Calculator Problem 1: Two-Tailed Test The CEO of a large electric utility claims that 80 percent of his 1,000,000 customers are very satisfied with the service they receive. To test this claim, the local newspaper surveyed 100 customers, using simple random sampling. Among the sampled customers, 73 percent say they are very satisified. Based on these findings, can we reject the CEO's hypothesis that 80% of the customers are very satisfied? Use a 0.05 level of significance. Solution: The solution to this problem takes five steps: (1) state the hypotheses, (2) choose the significance level, (3) compute the test statistic, (4) find the P-value, and (5) interpret results. We work through those steps below: State the hypotheses. The first step is to state the null hypothesis and an alternative hypothesis. Null hypothesis: P = 0.80 Alternative hypothesis: P ≠ 0.80 Note that these hypotheses constitute a two-tailed test. The null hypothesis will be rejected if the sample proportion is too big or if it is too small. Choose the significance level. For this analysis, the significance level is 0.05. Compute the test statistic. The one-sample z-test is an appropriate method to determine whether a hypothesized population proportion differs significantly from an observed sample proportion. The test statistic for a one-sample z-test is a z-score. We calculate the standard deviation (SD) and compute the z-score test statistic (z) from the formulas below: SD = sqrt[P0 * (1 - P0) / n] SD = sqrt [(0.8 * 0.2) / 100] SD = sqrt(0.0016) = 0.04 z = (p - P0) / SD = (.73 - .80)/0.04 = -1.75 where P0 is the hypothesized value of population proportion in the null hypothesis, p is the sample proportion, and n is the sample size. Find the P-value. Since we have a two-tailed test, the P-value is the probability of observing a z-score more extreme than the absolute value of the test statistic (i.e., less than -1.75 or greater than 1.75). We use the Normal Distribution Calculator to find P(z < -1.75) = 0.04. Since the standard normal distribution is symmetric with a mean of zero, we know that P(z > 1.75) = 0.04. Thus, the P-value = 0.04 + 0.04 = 0.08. Interpret results. Since the P-value (0.08) is greater than the significance level (0.05), we cannot reject the null hypothesis. Note: If you use this approach on an exam, you may also want to mention why this approach is appropriate. Specifically, the approach is appropriate because the sampling method was simple random sampling, the sample included at least 10 successes and 10 failures, and the population size was at least 10 times the sample size. Problem 2: One-Tailed Test Suppose the previous example is stated a little bit differently. Suppose the CEO claims that at least 80 percent of the company's 1,000,000 customers are very satisfied. Again, 100 customers are surveyed using simple random sampling. The result: 73 percent are very satisfied. Based on these results, should we accept or reject the CEO's hypothesis? Assume a significance level of 0.05. Solution: The solution to this problem takes five steps: (1) state the hypotheses, (2) choose the significance level, (3) compute the test statistic, (4) find the P-value, and (5) interpret results. We work through those steps below: State the hypotheses. The first step is to state the null hypothesis and an alternative hypothesis. Null hypothesis: P = 0.80 Alternative hypothesis: P < 0.80 Note that these hypotheses constitute a one-tailed test. The null hypothesis will be rejected only if the sample proportion is too small. Choose the significance level. For this analysis, the significance level is 0.05. Compute the test statistic. The one-sample z-test is an appropriate method to determine whether a hypothesized population proportion differs significantly from an observed sample proportion. The test statistic for a one-sample z-test is a z-score. We calculate the standard deviation (SD) and compute the z-score test statistic (z) from the formulas below: SD = sqrt[P * (1 - P0) / n] = sqrt [(0.8 * 0.2) / 100] SD = sqrt(0.0016) = 0.04 z = (p - P0) / SD = (.73 - .80)/0.04 = -1.75 where P0 is the hypothesized value of population proportion in the null hypothesis, p is the sample proportion, and n is the sample size. Find the P-value. Since we have a one-tailed test , the P-value is the probability that the z-score is less than -1.75. In the previous problem, we used the Normal Distribution Calculator to find P(z ≤ -1.75) = 0.04. Thus, the P-value = 0.04. Interpret results. Since the P-value (0.04) is less than the significance level (0.05), we cannot accept the null hypothesis. Problem 3 In Problem 1 and Problem 2 above, we used identical sampling plans (same sampling method, same sample size, etc.), the null hypothesis (H0 = 0.80) was the same in both problems, and the P-value (-1.75) was the same in both problems. We rejected the null hypothesis in Problem 2 but not in Problem 1. Explain how this happened. Solution: In both problems, the significance level was 0.05. But we conducted a two-tailed test in Problem 1 and a one-tailed test in problem 2. That makes a difference. Two-tailed test. The area of rejection is split between the two tails of the sampling distribution. Since the significance level was 0.05, 2.5% of the rejection area would be in each tail, and if the test statistic falls in either of these small tails, we reject the null hypothesis. One-tailed test. The rejection area is only on one side of the sampling distribution. So, all 5% of the rejection area is placed in the lower tail. If the test statistic falls in this larger area, we reject the null hypothesis. Basically, the region of rejection in the lower tail was larger for the one-tailed test than for the two-tailed test. That allowed us to reject the null hypothesis in Problem 2, but not in Problem 1. If you would like to cite this web page, you can use the following text: Berman H.G., "Hypothesis Test for a Proportion", [online] Available at: URL [Accessed Date: 5/17/2025]. A population proportion is the share of a population that belongs to a particular category. Hypothesis tests are used to check a claim about the size of that population proportion. Hypothesis Testing a Proportion The following steps are used for a hypothesis test: Check the conditions Define the claims Decide the significance level Calculate the test statistic Conclusion For example: Population: Nobel Prize winners Category: Born in the United States of America And we want to check the claim: "More than 20% of Nobel Prize winners were born in the US" By taking a sample of 40 randomly selected Nobel Prize winners we could find that: 10 out of 40 Nobel Prize winners in the sample were born in the US The sample proportion is then: \(\displaystyle \frac{10}{40} = 0.25\), or 25%. From this sample data we check the claim with the steps below. 1. Checking the Conditions The conditions for calculating a confidence interval for a proportion are: The sample is randomly selected There is only two options: Being in the category Not being in the category The sample needs at least: 5 members in the category 5 members not in the category In our example, we randomly selected 10 people that were born in the US. The rest were not born in the US, so there are 30 in the other category. The conditions are fulfilled in this case. Note: It is possible to do a hypothesis test without having 5 of each category. But special adjustments need to be made. 2. Defining the Claims We need to define a null hypothesis (\(H_{0}\)) and an alternative hypothesis (\(H_{1}\)) based on the claim we are checking. The claim was: "More than 20% of Nobel Prize winners were born in the US" In this case, the parameter is the proportion of Nobel Prize winners born in the US (\(p\)). The null and alternative hypothesis are then: Null hypothesis: 20% of Nobel Prize winners were born in the US. Alternative hypothesis: More than 20% of Nobel Prize winners were born in the US. Which can be expressed with symbols as: \(H_{0}\): \(p = 0.20 \) \(H_{1}\): \(p > 0.20 \) This is a 'right tailed' test, because the alternative hypothesis claims that the proportion is more than in the null hypothesis. If the data supports the alternative hypothesis, we reject the null hypothesis and accept the alternative hypothesis. The significance level (\(\alpha\)) is the uncertainty we accept when rejecting the null hypothesis in a hypothesis test. The significance level is a percentage probability of accidentally making the wrong conclusion. Typical significance levels are: \(\alpha = 0.1\) (10%) \(\alpha = 0.05\) (5%) \(\alpha = 0.01\) (1%) A lower significance level means that the evidence in the data needs to be stronger to reject the null hypothesis. There is no "correct" significance level - it only states the uncertainty of the conclusion. Note: A 5% significance level means that when we reject a null hypothesis: We expect to reject a true null hypothesis 5 out of 100 times. 4. Calculating the Test Statistic The test statistic is used to decide the outcome of the hypothesis test. The test statistic is a standardized value calculated from the sample. The formula for the test statistic (TS) of a population proportion is: \(\displaystyle \frac{\hat{p} - p}{\sqrt{p(1-p)}} \cdot \sqrt{n} \) \(\hat{p}-p\) is the difference between the sample proportion (\(\hat{p}\)) and the claimed population proportion (\(p\)). \(n\) is the sample size. In our example: The claimed (\(H_{0}\)) population proportion (\(p\)) was \(0.20 \) The sample proportion (\(\hat{p}\)) was 10 out of 40, or: \(\displaystyle \frac{10}{40} = 0.25\) The sample size (\(n\)) was \(40\) So the test statistic (TS) is then: \(\displaystyle \frac{0.25-0.20}{\sqrt{0.2(1-0.2)}} \cdot \sqrt{40} = \frac{0.05}{\sqrt{0.2(0.8)}} \cdot \sqrt{40} = \frac{0.05}{\sqrt{0.16}} \cdot \sqrt{40} \approx \frac{0.05}{0.4} \cdot 6.325 = \underline{0.791}\) You can also calculate the test statistic using programming language functions: With Python use the scipy and math libraries to calculate the test statistic for a proportion. import scipy.stats as stats import math # Specify the number of occurrences (x), the sample size (n), and the proportion claimed in the null-hypothesis (p) x = 10 n = 40 p = 0.2 # Calculate the sample proportion p_hat = x/n # Calculate and print the test statistic print((p_hat-p)/(math.sqrt((p*(1-p))/(n)))) Try it Yourself » With R use the built-in prop.test() function to calculate the test statistic for a proportion. # Specify the sample occurrences (x), the sample size (n), and the null-hypothesis claim (p) x